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Investigating the Students' Mathematical Fluency and Procedural Skills using P-cube (P³) as an Authentic Assessment

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ABSTRACT

Mathematical fluency and procedural skills are essential for students to deepen their understanding of mathematics and apply knowledge to real-world situations. However, the 2022 PISA results revealed that the Philippines continues to perform poorly in mathematics, underscoring the need to strengthen these competencies among students. To address this, the researcher developed the P-cube(P³) assessment approach, which stands for Practice, Participate, and Portfolio. This study implemented P³ as an authentic assessment tool to describe the mathematical fluency and procedural skills of Grade 10-Astudents at Libertad National High School and to explore their perceptions of the assessment. Results indicated positive student responses, with only a few negative remarks. Procedural skills, measured through student scores, clustered around the mean, and the standard deviation was within acceptable limits, suggesting consistent performance levels. Few extreme scores were recorded. Interviews further revealed that students found mathematics more understandable and enjoyable through interactive and engaging activities. Although they encountered challenges such as multiple drills, extensive hands-on tasks, and complex problems, they reported improvements in problem-solving, communication, and creativity. These qualitative results aligned with strong performance on P-cubeactivities, though the study did not track improvement over time. Overall, the P-cubeassessment proved engaging and effective in promoting mathematical fluency and procedural understanding. Students expressed that learning mathematics—particularly topics like polynomial functions, became more enjoyable through this authentic, activity-based approach, making P³ a promising model for future classroom implementation.

Key Words: Authentic Assessment, Procedural Skills, Mathematical Fluency, Mathematics Assessment, Mathematics Teaching.

1. INTRODUCTION

Mathematics is a fundamental discipline essential for developing problem-solving and logical reasoning abilities, which are crucial in everyday life and across academic domains. It provides the cognitive foundation for understanding the natural world and underpins many professional fields. Despite its importance, students worldwide often perceive mathematics as difficult and abstract, which affects their motivation and performance (Gafoor & Kurukkan, 2015). Strengthening mathematical fluency and procedural skills is therefore a central aim of mathematics education, as these two competencies form the basis for effective problem-solving and application of mathematical knowledge in authentic contexts.

According to Cartwright (2020), mathematical fluency emerges from the interaction between strategy use, reasoning ability, and conceptual understanding. It involves not merely memorization of formulas and procedures but the ability to flexibly and efficiently apply mathematical concepts to solve problems. Complementarily, procedural skills refer to the correct selection and execution of algorithms and methods, as well as the ability to verify and adapt these processes when confronted with varied problem situations (Hrmo & Gonda, 2021). When fluency and procedural skills develop together, students attain deeper understanding and the capability to apply mathematical reasoning in real-world problems (Ncube & Luneta, 2025).

International assessments reveal the continuing challenge of developing these skills among students. The Program for International Student Assessment (PISA) evaluates 15-year-old students' ability to formulate, interpret, and apply mathematics in diverse contexts (Ekmekci, 2013). Results from the 2022 PISA cycle indicate that Filipino learners lag by about five to six years in mathematical competencies, ranking in the lower tier among 81 participating nations (Servallos, 2023; Palasi, 2025). This performance gap underscores an urgent need to re-examine mathematical teaching and assessment practices in the Philippines.

Teaching mathematics is inherently challenging because it requires higher-order thinking and extensive abstract reasoning. Many students find it unengaging, often associating it with rote memorization rather than exploratory understanding (Dimatacot & Parangat, 2022; Valerio, 2015). However, mathematics remains one of the most vital subjects in the school curriculum due to its key role in critical thinking and scientific literacy (Crowe, 2022; Fitzmaurice, O'Meara, & Johnson, 2021; Hojjat, Mohsen, Javad, & Ghasem, 2015). The Mathematics Framework for Philippine Basic Education developed by DOST-SEI and MATHTED highlights that learning cannot occur effectively through teacher-centered methods alone; learners must be actively involved in constructing meaning (Olazo, 2019; Ghafar, 2023; Woods, Copur-Gencturk, Woods, & Copur-Gencturk, 2023). Lanya, Susiswo, Hidayanto, and Rahardjo (2024) further note that active participation—questioning, reasoning, and debating—optimizes learning potential and develops a richer appreciation of mathematics.

1.1 The P-cube(P^3) Assessment Framework

In response to this pedagogical challenge, the researcher conceptualized the P-cube(P^3) framework, which stands for Practice, Participate, and Portfolio. P^3 is designed as an authentic, classroom based assessment system that integrates teaching, learning, and assessment. It operationalizes Vygotsky's sociocultural learning theory and aligns with the National Council of Teachers of Mathematics (NCTM) emphasis on procedural fluency and mathematical discourse (Blankman, 2024). Rather than proposing a new learning theory, P^3 functions as a practical assessment model that measures students' ongoing development of mathematical fluency and procedural skills through iterative, participatory, and reflective learning processes.

The Practice component centers on structured, teacher-guided individual activities that provide varied opportunities to apply mathematical procedures in progressively complex tasks. In this study, practice was implemented through daily drills and written exercises on polynomial functions. These exercises encouraged recall, procedural accuracy, and conceptual reinforcement, consistent with research emphasizing deliberate practice as foundational to computational fluency (Al-Mutawah, Thomas, Eid, Mahmoud, & Fateel, 2019). Drawing on Vygotsky's concept of the Zone of Proximal Development (ZPD), scaffolded practice tasks enable learners to transition from basic to more complex problems with teacher guidance (Helwig-Henseleit, 2025). Consequently, the Practice component improves proficiency, efficiency, and confidence through repeated engagement and immediate corrective feedback (Grønli, Walgermo, Uppstad, & McTigue, 2025).

The Participate component promotes collaborative learning and mathematical communication. In line with sociocultural principles, students work in small groups to solve contextualized problems, explain reasoning, and critique each other's solutions. Such interaction situates learning within social discourse, promoting understanding of when and why specific procedures are appropriate (Esmonde & Langer-Osuna, 2013; Cartwright, 2020). Vygotsky posited that knowledge is co-constructed through dialogue with peers and experts, making social engagement central to cognitive growth. By articulating thought processes and evaluating multiple strategies, students develop procedural flexibility, reasoning precision, and confidence in problem-solving (Hassan, 2024).

The final component, Portfolio, emphasizes reflection and consolidation. Students compile their mathematical outputs-including problem solutions, graphical analyses, and group activity results—into an individualized portfolio that serves as both a summative and formative assessment tool. Portfolio assessment enables learners and teachers to monitor progress, identify misconceptions, and recognize growth in fluency and procedural competence over time. This process supports metacognitive awareness, allowing learners to evaluate their strategies and adjust learning approaches (Lukitasari, Hasan, Sukri, & Handhika, 2021). Portfolios also encourage creativity and ownership of

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learning by showcasing the students' evolving mathematical understanding in open-ended and problem-based tasks (Vale & Barbosa, 2023).

Together, these three components form an integrated assessment cycle. Practice ensures mastery through repetition; Participate strengthens reasoning through collaboration; and Portfolio fosters reflection and metacognition. Thus, P^3 aligns with NCTM's definition of procedural fluency as involving accuracy, efficiency, and flexibility—beyond rote learning (Andal & Andrade, 2022). Empirical data from the study, including high mean activity scores and positive student reflections, suggest that the P^3 approach enhances problem-solving, communication, and creativity. These findings reinforce that authentic, socially grounded assessment fosters deeper mathematical engagement, consistent with the theoretical foundation of Vygotsky (1978) and Cartwright's (2020) work on fluency and reasoning.

1.2 Context and Rationale of the Study

The implementation of P^3 took place in Libertad National High School, Misamis Oriental. Like many public schools in the Philippines, the institution faces challenges related to limited instructional materials and insufficient localized resources for effective mathematics teaching (Lopez Jr. & Roble, 2022). Traditional materials often fail to align with active, student-centered approaches, constraining opportunities for applied learning. The P^3 framework was thus introduced as a context-responsive strategy for improving mathematical fluency and procedural skills among Grade 10 students through authentic assessment practices integrated with classroom instruction.

1.3 Theoretical Framework

The P^3 framework is anchored in Vygotsky's sociocultural theory, which views learning as a culturally mediated, socially situated process (Vygotsky, 1978). Schools function as environments where learners internalize knowledge through interaction and shared activity with teachers and peers. Central to this perspective is the Zone of Proximal Development (ZPD)—the gap between what learners can achieve independently and what they can achieve with guided support (Chaiklin, 2013). As learners engage in scaffolded practice, group participation, and reflective documentation, they internalize cultural tools such as mathematical language, symbols, and procedures (Esparcia, Piñero, & Futalan, 2024). Within this dynamic, feedback, modeling, and collaboration serve as scaffolds that enable learners to progressively achieve greater competence, thus improving mathematical fluency and procedural mastery (Zhou, 2024).

In Vygotskian terms, P^3 transforms assessment from a fixed evaluation of outcomes into a mediated learning process. Each component of P^3 maps onto the key sociocultural dimensions of learning: Practice corresponds to scaffolding within the ZPD, Participate reflects the social construction of knowledge through discourse, and Portfolio represents internalization and self-regulation. Together, they embody a holistic approach that connects assessment with teaching and learning in real-time contexts. By situating learning in authentic problem-solving and collaborative interactions, P^3 aligns with contemporary research advocating authentic assessment as a means to capture the complexity of students' mathematical reasoning (Nardo, 2021; McLeod, 2025).

Ultimately, the P-cube(P^3) Assessment Framework redefines mathematics assessment as a dynamic, participatory process rather than a static measurement of competence. By systematically integrating practice, collaboration, and reflection, it cultivates essential mathematical skills while addressing gaps identified in national and international evaluations.

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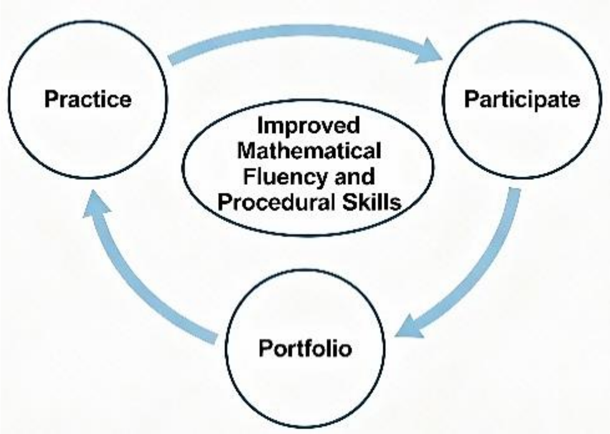

Figure 1. Conceptual framework of the P-cube(P³) assessment method

The framework's alignment with sociocultural principles and national curriculum goals provides a theoretically grounded and practically viable model for improving mathematics instruction and learner outcomes in the Philippine context.

Figure 1 illustrates P-cube (P³) as a cyclical assessment method in which Practice, Participate, and Portfolio continually reinforce one another to enhance students' mathematical fluency and procedural skills. The Practice component draws from research on procedural fluency and feedback-supported rehearsal. The National Council of Teachers of Mathematics (2014, as cited in Blankman, 2024) defines procedural fluency as the ability to apply procedures flexibly, accurately, and efficiently, emphasizing conceptual understanding over memorization. Studies by Al-Mutawah, Thomas, Eid, Mahmoud, and Fateel (2019) and Graven and Stott (2012) demonstrate that repeated and varied engagement, supported by timely feedback and opportunities for self-correction, improves both accuracy and adaptability. In this study, Practice involves tasks on polynomial functions that require students to recall formulas, choose appropriate procedures, and validate results with teacher scaffolding. Explanations, worked examples, and feedback act as supports within the Zone of Proximal Development (ZPD) (Vygotsky, 1978; Main, 2023), promoting gradual internalization of procedural knowledge (Azi, 2020).

The Participate component emphasizes sociocultural and discourse-based approaches to mathematical learning. Based on Vygotsky's (1978) theory, cited by Li (2023), interactions with peers and teachers nurture higher-order thinking. Supporting research shows that classroom discourse plays a vital role in this process. Esmonde and Langer-Osuna (2013) describe how participation structures influence access to mathematical ideas; Ehrenfeld and Horn (2020) highlight the teacher's role in scaffolding group discussions; Hintz and Tyson (2015) propose complex listening to advance sense-making; while Cartwright (2020) shows how communication improves procedural understanding. The Participate phase in P³ (Lanya, Susiswo, Hidayanto, & Rahardjo, 2024) involves hands-on, small-group activities where students justify, argue, and negotiate mathematical reasoning. Through these exchanges, learners refine strategies, develop confidence, and expand procedural flexibility.

The Portfolio component aligns with research on authentic assessment and self-regulated learning, positioning students as active participants in monitoring their growth (Bures, Barclay, Abrami, & Meyer, 2013). Hudesman, Crosby, Flugman, Issac, and Everson (2020) emphasize that formative assessment should promote reflection and metacognition rather than classification. Portfolios in mathematics capture students' problem-solving strategies, encouraging analysis and self-evaluation (Šliogerienė, 2016; Sbhatu & Weldeana, 2017). Within P³, students compile evidence from Practice and Participate—including written solutions, reflections, and group artifacts—to demonstrate progress. By reviewing earlier work and comparing it with recent achievements, learners engage in reflective monitoring consistent with Gudeta (2022).

Collectively, these interdependent components embody the theoretical foundation of P-cube (P³) as an authentic, learner-centered evaluation framework. Practice derives from NCTM's (2014) conceptualization of

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procedural fluency and research on feedback-based rehearsal. Participate operationalizes Vygotsky's notion of social mediation alongside contemporary findings on mathematical discourse and group collaboration (Topçiu & Myftiu, 2015). Portfolio builds on formative assessment traditions emphasizing reflection and self-regulated learning (Alkaabi & Abdallah, 2024). Together, these components create a socially mediated cycle where guided individual practice develops competence, collaborative participation deepens understanding, and reflective portfolios consolidate learning.

In this study, P³ is implemented as a classroom-based assessment framework grounded in the sociocultural theory of learning. Practice provides scaffolded opportunities to apply procedures (Hofer & Reinhold, 2025). Participate engages students in hands-on, small-group problem solving that embeds procedural mastery within discourse (Anderson-Pence, 2017). Portfolio documents progress through organized student work and reflections, allowing both teacher and student to trace growth in fluency (Sulistyo, Eltris, Mafulah, Budianto, Saiful, & Heriyawati, 2020). This cyclical model uses scaffolding and discourse within the ZPD to support the transition from guided practice to independent understanding (Alber, 2014; Helwig-Henseleit, 2025). Although no causal claims can be made, quantitative and qualitative findings indicate high levels of procedural fluency and performance throughout P³ implementation, affirming its potential as a valid, sociocultural grounded assessment approach.

2. OBJECTIVES OF THE STUDY

This study investigates the mathematical fluency and procedural skills of Grade 10-Astudents at Libertad National High School following exposure to the P-cube(P³) assessment. Specifically, it seeks to determine: (1) the students' levels of mathematical fluency under the P-cubemethod, and (2) their levels of procedural skills in mathematics when evaluated through the same assessment framework.

3. METHODOLOGY

This study employed an explanatory sequential mixed-methods design, beginning with quantitative analysis of student scores from Practice, Participate, and Portfolio activities using descriptive statistics to gauge mathematical fluency and procedural skills. Results guided the selection of participants for semi-structured interviews, which explored students' experiences with the P-cube(P³) assessment and clarified quantitative findings. Integration occurred during interpretation, where qualitative themes were used to explain and contextualize score distributions.

3.1 Participants of the Study

Using purposive sampling, one intact Grade 10 mathematics class (N = 43) from Libertad National High School was chosen, as administrative policies required applying the full P-cube(P³) sequence to an existing section. All students provided consent or assent, resulting in 100% participation without missing cases (Wakefield, 2000). Since the sample was non-random and drawn from a single school, the results are context-specific rather than statistically generalizable (Taherdoost, 2016).

3.2 Data Gathering Methods and Instruments

This study utilized an explanatory sequential mixed-methods design, beginning with quantitative analysis of P-cube(P³) performance and conventional test scores to identify patterns in mathematical fluency and procedural skills. These results guided sampling and variable focus for the subsequent qualitative phase (Shaheen, Pradhan, & Ranajee, 2018). The qualitative component, composed of interview transcripts, written reflections, and task artifacts, explored how students experienced and interpreted the P³ assessment, linking their narratives to observed quantitative trends to form integrated meta-inferences on how P³ fosters fluency and procedural proficiency (Apuke, 2017). Integration occurred during analysis and interpretation through cross-strand case linkages, comparison of quantitative indicators with emergent qualitative themes, and construction of a joint display illustrating the mixed-methods logic (Shaw, Hiles, West, Holland, & Gwyther, 2018).

At the time of data collection, Libertad National High School lacked a dedicated ethics review board; thus, the study followed Department of Education protocols, with approvals from the Schools Division Superintendent, District Supervisor, Principal, and Mathematics Department Head. Although no formal IRB number was issued, the research complied with ethical principles of respect, beneficence, and justice, ensuring voluntary participation with informed

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consent and assent from all 43 students and guardians. Confidentiality was maintained using respondent codes (R1–R34) and secure data storage.

Conducted over two weeks on polynomial functions, the implementation included Practice drills, Participate group tasks, and Portfolio documentation. The student questionnaire, adapted from TIMSS 2007, achieved high reliability ($\alpha = 0.85$ for fluency; $\alpha = 0.81$ for procedural skills) following validation by content experts.

3.3 Data Analysis and Statistical Treatment

Quantitative and qualitative data were analyzed concurrently to generate complementary evidence about students' mathematical fluency, procedural skills, and experiences with the P-cube (P^3) assessment. In line with an explanatory sequential mixed-methods design, the quantitative and qualitative strands were integrated during the interpretation phase, where qualitative themes were used to explain and contextualize the score distributions (Dempsey, 2018).

To establish content and face validity, an initial pool of items was evaluated by a mathematics education faculty member, a high school mathematics department head, and an experienced Grade 10 mathematics teacher, focusing on relevance, clarity, and alignment with the DepEd Grade 10 mathematics curriculum. Minor revisions were made to simplify wording, remove overlapping content, and ensure that each item clearly represented either mathematical fluency or procedural skills, resulting in two 10-item scales. A small pilot administration with a comparable Grade 10 class in the same school was then conducted to check comprehension, response use, and completion time; no substantial issues were identified, so the refined instrument was used with the Grade 10-A class in the main study.

Internal consistency reliability of the adapted scales was examined using Cronbach's alpha computed (Taber, 2017). from the responses of the 30 Grade 10-E students in the same school, with values at or above the commonly cited threshold of 0.70 interpreted as acceptable for research purposes. The 10-item mathematical fluency scale produced a Cronbach's alpha of $\alpha = 0.85$, and the 10-item procedural skills scale produced a Cronbach's alpha of $\alpha = 0.81$, both indicating acceptable internal consistency. Inspection of corrected item–total correlations showed that no item would substantially increase alpha if deleted, so all items were retained.

The questionnaire used a 4-point response format (4 = every or almost every lesson, 3 = about half of the lessons, 2 = some lessons, 1 = never) (Siegle, 2015), and item responses were entered and processed using Microsoft Excel. For each item and for each scale, mean, median, mode, and overall mean scores were computed, and frequency distributions were generated to categorize students' levels of mathematical fluency and procedural skills when exposed to the P-cube (P^3) assessment. Descriptive statistics for practice and participation activity scores were likewise calculated to illuminate the level and variability of students' engagement with the assessment activities (Kalobo, 2025).

The study followed a one-group post-implementation design in which the P-cube (P^3) assessment was purposefully embedded into regular instruction for two weeks, with data collected at the end of the unit (Smith, 2023). Because no baseline questionnaire or test scores were obtained for the same class, the quantitative analysis was not intended to estimate pre–post effect sizes but rather to provide a detailed descriptive profile of students' fluency, procedural skills, and participation under full implementation. Within this design, the quantitative findings were interpreted as evidence of students' generally high levels of fluency and procedural skills and positive reported experiences with P-cube (P^3), while recognizing that more rigorous designs such as, pre–post or comparison-group studies would be required to support causal claims about improvement over time (Byers, 2016). Qualitative data from the open-ended questionnaire items and follow-up interviews was analyzed using Braun and Clarke's reflexive thematic analysis approach, proceeding through familiarization, coding, theme development, and refinement (Naeem, Ozuem, Howell, & Ranfagni, 2023). First, all responses and interview transcripts were read multiple times and lightly annotated to gain an overall sense of students' experiences with the P-cube (P^3) assessment. Second, initial codes were generated inductively at the semantic level, with short labels capturing features such as perceived benefits, specific challenges, emotional reactions, and reported changes in engagement or understanding (H, 2025). Coding was conducted using a combined manual and spreadsheet-based system, allowing codes to be attached to line-by-line segments of text (Linneberg, & Korsgaard, 2019).

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Third, related codes were clustered into candidate themes such as “making mathematics easier and more enjoyable,” “productive struggle with complex tasks,” and “developing problem-solving and communication skills”, which were then iteratively reviewed against the coded extracts and the full data set to ensure internal coherence and clear distinctions between themes. Themes were refined by collapsing overlapping categories, splitting overly broad themes, and checking that each theme directly addressed the research questions about students’ views of the P-cube (P^3) assessment (Kiger, & Varpio, 2020). To enhance trustworthiness, a second mathematics education researcher independently coded a subset of 25–30% of the data using the preliminary codebook (Halpin, 2024). Inter-coder agreement on code application for this subset was calculated using percentage agreement, discussed in terms of consistency rather than as a rigid reliability coefficient, and discrepancies were resolved through discussion, leading to further clarification of code definitions and minor revisions to theme boundaries. Throughout the process, an audit trail of coding decisions, codebook iterations, and theme changes was maintained, and representative quotations were selected to illustrate each final theme in the Findings section.

Finally, quantitative and qualitative results were brought together at the interpretation stage through side-by-side comparison of questionnaire scores, activity means, and interview themes, with attention to areas of convergence, where both strands told a similar story, complementarity, where one strand elaborated the other, and divergence, where the strands suggested different emphases (Almalki, 2016).

4. FINDINGS AND DISCUSSION

After implementing the P-cube (P^3) assessment method in Grade 10-A of Libertad National High School, the mathematical fluency and procedural skills were measured through survey questionnaire modified from the student questionnaire of the Trends in International Mathematics and Science Study (TIMSS) (2007) (Stanco, 2015).

4.1 Research Question 1: What are the levels of mathematical fluency of the Grade 10-Astudents when exposed to the P-cubeassessment method?

Table 1 shows the mathematical fluency of the Grade 10-A students of Libertad National High School. There are 10 statements in the questionnaire. Each of the students answered the questionnaire by checking their answers if the statement about mathematics is experienced every or almost every lesson, about half of the lessons, some lessons, none of the lessons.

Table 1 shows that the Grade 10-A students responded positively on most mathematical fluency items after being exposed to the P-cube assessment. All items had many responses in the ‘every or almost every lesson’ category, indicating that these activities occurred frequently in their lessons. The statement with the highest number of students who answered the “every or almost ever lesson” is the memorization of formulas and procedures. According to Luu, (2024), mathematical fluency is the ability to quickly recall simple formulas and procedures in mathematics. Students can learn facts based on conceptual learning, fact strategies, and memorization. It is supported by the research of Östergren, Träff, Elofsson, Hesser, & Samuelsson (2023). which stated that using memorization training can improve students’ fluency in mathematics.

Majority of the students also agreed to the statement that they work in small groups in every or almost every lesson they had in Polynomial Functions. Students can communicate more than just the basic mathematical fluency when expressing their solutions and strategies in groups (Graven, & Stott, 2015). According to the Common Core State Standards Initiative, communicating between students is a part of their ability to justify their conclusions, communicate them to their classmates and teachers, and respond to the arguments of others. Mathematics must be communicated (Wakefield, 2000).

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Table 1
Mathematical Fluency of Grade 10-A Students after the Implementation of P-cube (P³) Assessment Method

Statements	Frequency				Total
	4	3	2	1	
1. I practice adding, subtracting, multiplying, and dividing without using a calculator.	30	7	6	0	43
2. I work on Polynomial Functions drills and hands-on activities.	33	7	3	0	43
3. I solve problems about Polynomial Functions.	33	8	2	0	43
4. I interpret data in tables, charts, or graphs.	26	10	7	0	43
5. I write equations and functions to represent relationships.	23	12	8	0	43
6. I memorize formulas and procedures.	39	4	0	0	43
7. I can explain my answers.	28	10	5	0	43
8. I review my homework.	25	15	3	0	43
9. I work together in small groups.	37	6	0	0	43
10. I know how to explain my outputs to the class.	20	9	14	0	43

On the other hand, according to Alsaleh (2020), students seem to agree in the statement that they know how to explain their outputs in some lessons only. It means that some of the students can explain what their outputs mean in selected topics only in Polynomial Functions. Some students reported having difficulty explaining their answers, particularly when connecting polynomial functions to real-world contexts, which made it challenging for them to articulate the underlying concepts.

The results indicate that, under the P-cube (P³) assessment, students reported high levels of mathematical fluency on most items, with the majority selecting 'every or almost every lesson' for core fluency behaviors.

These quantitative patterns are consistent with students' interview reports that practice and participation activities made the lessons "easier to understand" and helped them remember procedures and concepts, suggesting convergence between self-reported frequencies of fluency behaviors and students' narrative accounts of how P-cube supported their mathematical work (Valerio, 2015).

4.2 Research Question 2: What are the levels of procedural skills of the Grade 10-Astudents in mathematics when exposed to the P-cubeassessment method?

Another aspect examined in this study is students' procedural skills in mathematics. Understanding procedures is essential in learning mathematics because the subject places strong emphasis on problem solving. Table 2 shows the result of the Grade 10-A students' answers to the questionnaire on procedural skills.

Table 2 shows the procedural skills of the students when it comes to mathematics. Variety of answers from them was gathered. Procedural skills can be developed alongside conceptual understanding and reasoning. Students are able to relate the procedures to different contexts and problems (Thoe, Jamaludin, Pang, Choong, Lay, Ong, et. Al.,2022). In the answers of the students above, the highest frequency when it comes to activities being experienced in every lesson or almost every lesson is the solving of problems about Polynomial Functions through following the given process. Students were able to answer the given problems through the process stated in the book or discussed by their teacher. Following the formula and process are part of the problem-solving steps in mathematics (Al-Mutawah, Thomas, Eid, Mahmoud, & Fateel, 2019).

Another important observation is the presence of 'never' responses, indicating that some students did not experience certain procedural behaviors described in the items. The highest frequency under "never" is deciding on their own procedures for solving complex problems. It is an opposite statement with the highest frequency in "every lesson or almost every lesson". It only suggests that some of the students cannot think on their own procedures in

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solving the given problems about Polynomial Functions. They relied on the given process or procedure by the teacher. Procedural skill is the ability to modify procedures in order to deal with factors that can exist in problem settings (Cartwright, 2018).

However, some students did not demonstrate this procedural skill. Other students were also not confident in using procedures in solving mathematical problems. Self-confidence can make the students improve their mathematical thinking and believe that problems can be solved with passion and hard work to learn continuously in order to improve their abilities (Yaniawati, Kariadinata, Sari, Pramiarsih, & Mariani, 2020). Aiyub, Suryadi, Fatimah, & Kusnandi (2024) suggested that mathematical thinking starts with the given statements or processes to train how to resolve mathematical problems. Though, it is better for the students to think on another statement or process rather than just accept what was given by the teacher.

However, with the low of self-confidence in some students, majority of them also ask questions when confused with the process or procedure. Asking questions, students are engaged to process their knowledge, discover and even develop their own concepts of the topic. It provides an opportunity for students to enhance their higher order thinking skills (Hendriana, Putra, & Hidayat, 2019).

Table 2
Procedural Skills in Mathematics of Grade 10-A Students after the Implementation of P-cube (P^3) Assessment Method

Statements	Frequency				Total
	4	**3**	**2**	**1**	
1. I relate what we are learning in mathematics to our daily lives.	31	8	4	0	43
2. I decide on my own procedures for solving complex problems.	18	9	10	6	43
3. I work problems on my own.	30	11	2	0	43
4. I can select the correct formula or process in answering problems in the topic of Polynomial Functions.	35	5	3	0	43
5. I tend to verify the correctness of the procedure I use in answering problems.	38	5	0	0	43
6. I use concrete models in verifying the processes or procedures I use in a problem.	23	12	8	0	43
7. If the procedure I use is wrong, I have the tendency to modify this procedure into something correct.	20	10	10	3	43
8. I can solve problems about Polynomial Functions through following the given process.	39	4	0	0	43
9. I ask questions if I am confused with the process or procedure.	40	3	0	0	43
10. I am confident in using procedures in solving mathematical problems.	23	9	9	2	43

Likewise, students' descriptions of developing problem-solving, communication, and creativity skills during group tasks and portfolio work complement the procedural-skills scores, indicating that the high levels of procedural performance observed in drills and group activities were accompanied by perceived growth in how they approached and communicated about mathematical problems.

The results show that students in Grade 10-A of Libertad National High School demonstrated generally high procedural skills, as reflected in mean scores close to the maximum and few 'never' responses across items. Within the limits of the one-group post-implementation design, these findings indicate that students displayed strong

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procedural skills during the P-cube unit, but they do not provide direct evidence of change over time (Bierer, Dallaghan, Borges, Brondfield, Fung, & Huggett, et. al., 2025).

To investigate the procedural skills of the students, their scores were tabulated in both "Practice" and "Participate" activities. Mean scores were calculated to present how much students learn about the subject through drills and group activities. Their procedural skills were tested through series of drills as well as activities that stimulated group collaboration. Table 3 shows the five (5) activities with its total scores and the mean scores of the students.

Table 3 below presents the mean scores of the Grade 10-A students in each activity as well as their overall mean score. The result shows that in every activity, their mean scores are closer to the total scores. It only means their procedural skills are high. For the standard deviation, scores from activities 1 and 2 are interpreted as acceptable as they are closer to the mean. The values are relatively consistent. As for activities 3, 4, and 5, their standard deviation values are slightly higher which mean that some of the scores in these activities are extremely high or low. Data points are further away from the mean. However, these standard deviation values are still acceptable. Some parts of the test may be required to revise or investigate (Omda, & Sergent, 2024).

Table 3
Scores of Grade-10 Students during "Practice" Activities

Activities	Total Scores	Mean Scores	SD
1	5	4.07	0.86
2	5	4.05	0.90
3	25	22.09	1.95
4	15	13.00	1.75
5	10	8.05	1.33
Overall Score	60	51.26	5.86

Because the mean scores for the practice activities were close to the maximum possible scores, students can be described as demonstrating high procedural performance during these tasks. The descriptive results suggest that, in the context of the P-cube implementation on polynomial functions, students were generally able to apply procedures accurately in drills and activities. However, in the absence of pre-implementation data or a comparison group, these scores cannot be interpreted as direct evidence that P-cube itself improved procedural skills over time. Practicing the concepts can develop their knowledge and skills in mathematics (Hurrell, 2021).

During the "Participate" activities, scores were gathered from the students. These activities involved group participation and collaboration when it comes to solving problems related to Polynomial Functions. Table 4 presents their scores in each activity with the mean and standard deviation.

Table 4 shows that their mean scores are closer to the scores in each activity. It interprets high in their procedural skills in solving different mathematical problems. For the standard deviation, activities 1 and 2 show values closer to the mean. It means that scores are acceptable and relatively consistent. The remaining activities have slightly higher standard deviation. It means that some scores in these activities are extremely high or low. The values are farther away from the mean. However, same with the previous data, these standard deviation values are still acceptable. Some investigation of the test may be required (Chaiklin, 2013).

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Table 4
Mean Scores of Grade-10 Students during "Participate" Activities

Activities	Total Scores	Mean Scores	SD
1	10	8.02	0.99
2	10	8.28	0.93
3	20	18.05	1.54
4	25	23.12	1.76
5	30	28.05	1.85
Overall Score	95	85.51	5.93

The presented data show that students obtained high mean scores across the participation activities, indicating strong procedural performance when solving problems in groups. These descriptive patterns suggest that, during the P-cubeimplementation, students were generally able to apply mathematical concepts collaboratively to solve polynomial problems. Given the one-group post-implementation design, these findings characterize students' procedural skills under P-cubebut do not demonstrate improvement relative to an earlier baseline.

Research Question 3: How do the Grade 10-A students view the P-cube (P^3) assessment method in Mathematics?

The last part of this intervention was the interview of the Grade 10-A students. After the series of practice, participation, and making of portfolio to showcase their outputs, students were asked series of open-ended questions to ensure more responses from the respondents. The meaning of the codes is the following: FS means female student and MS means male student. The number was their sequence when they were interviewed. Table 5 shows their answers of the interview questions.

Table 5 below shows the responses of the students during the interview. The interview has the main theme of perceptions of students on P-cube (P^3) assessment method. Based on the responses, they had learned a lot during the implementation of P-cube (P^3). P-cube (P^3) can make the lesson easy and make them remember the lesson more. In order to improve students' performance in a certain lesson, there should be analysis on the area where they find difficult.

It is the first step in the process of improving their performance especially in mathematics (Wijaya, Retnawati, Setyaningrum, Aoyama, & Sugiman, 2019). This study involves drills and hands-on activities of the mathematics lesson. Hands-on activity-based learning finds knowledge directly through the experiences of the students, constructing their own knowledge and understanding. In mathematics, learning is the same with hands-on activities and practical activities (Nurjanah, Jarnawi, & Wibisono, 2021). The responses suggest that mathematics topics can easily be learned through practice and participation. Students can easily grasp mathematical concepts through drills and hands-on activities.

Table 5
Summary of the Students' Perception on P-cube (P^3) Assessment Method

Theme	Coded For	Quote
Perception on P-cube (P^3) Assessment Method	Lessons Learned	**FS4:** "I learned that Polynomial Functions can be learned easily through the P-cube (P^3) assessment method." **MS15:** "I learned that mathematics can become bearable through the P-cube (P^3) assessment method." **MS26:** "I learned more about the Polynomial Functions." **FS32:** "I learned that we can improve our understanding in

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	mathematics through practice, participation, and submission of portfolio. It can make us remember more the lesson.”
Challenges Encountered	**MS1:** “Minsan mahirap ang drills and activities.”
	MS9: “I learned something about Poynomial Functions pero minsan marami kaming gagawin sa isang lesson.”
	FS7: “Mahirap minsan ang problems na binibigay at marami.”
Skills Developed	**FS41:** “It develops my knowledge in mathematics.”
	FS43: “P-cube (P^3) assessment develops my problem solving skills and communication skills dahil sa group activities.”
	MS5: “It develops my patience and perseverance dahil sa hirap intindihan ang ibang problems.”
	MS16: “P-cube (P^3) assessment develops my creativity skill dahil sa paggawa ng portfolio. Nadevelop din knowledge ko sa Math dahil sa daily drills and hands-on activities.”
	MS29: “Maganda ang P-cube (P^3) assessment kahit na maraming activities dahil nadedevelop ang knowledge sa Math especially and problem-solving skills.”
P-cube (P^3) importance	**FS3:** “It is important for me because I practiced my skills in mathematics every day.”
	FS11: “P-cube (P^3) is important because it can make the lesson fun and easy to understand.”
	MS8: “P-cube (P^3) is important because we are having fun while learning Math.”
	MS37: “It is important because it encourages us to solve problems na magiging helpful in the future.”

While it is true that P-cube (P^3) assessment method can help students in learning mathematics, they also had encountered challenges in this kind of method. One example mentioned is the difficulty of the problems given by the teacher.

Additionally, there are a lot of drills and activities that other students cannot cope with. These challenges can lead to mathematics anxiety. Mathematics anxiety can lead to poor performance of the students. They less motivated and confidence in taking the subject since they see that it is difficult and believe that they can’t solve the problems on their own or as members of the group (Namkung, Peng, & Lin, 2019). These current challenges and performance of the students can trigger them in developing mathematics anxiety in the future (Luo, Hogan, Tan, Kaur, Ng, & Chan, 2014).

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Skills were also developed in the implementation of the P-cube (P³) assessment method. Some of the mentioned skills are problem solving and communication skills. Moreover, creativity skill was mentioned due to the inclusion of the making of portfolio as an assessment method. It is supported by the study of (Siagian, Saragih, & Sinaga, 2019) as they used Problem-Based Learning. The mathematical problem-solving ability of students improved as they implemented the PBL in social arithmetic topic. Their results had shown that exposing students to different problems can improve their understanding and ability to apply mathematical concepts in everyday life.

When it comes to communication skills, the study of Tukaryanto (2015) suggested that discovery learning in mathematics can improve the students' communication skills. Learning mathematics involve communicating the students' thoughts well. The manner of teaching of the teacher can also affect the development of their communication skills in mathematics (Dina, Ikhsan, & Hajidin, 2019). Improvement of creativity skills can also be achieved in teaching mathematics. According to Chamberlin (2015), creativity in mathematics is an unusual ability.

It generates novel and important solutions of real-world problems using mathematical modelling. Posamentier, Smith, & Stepelman (2019) added that solving problems in mathematics is like inventing something new. Problem solving can promote creativity. Thus, students should be engaged with challenging problems.

Lastly, the P-cube (P³) importance was asked to the students. According to them, P-cube (P³) assessment method is important because they practiced their skills in mathematics every day. Additionally, this method made their discussions fun and engaging through the drills and hands-on activities. This particular scenario is helpful for them especially as they go to higher level of education. Their responses suggest that P-cube, as an authentic assessment method, helped them enjoy mathematics and perceive growth in several skills, although these perceptions were not verified with comparative baseline data. It may challenge them in some ways but it can be fun and engaging also (Çelik, 2019).

Taken together, the quantitative and qualitative strands provide a more complete picture of students' experiences with the P-cube (P³) assessment. Quantitatively, students reported high levels of mathematical fluency and procedural skills on most questionnaire items and obtained high mean scores on both practice and participation activities, indicating frequent engagement in core fluency behaviors and accurate application of procedures during the P-cube (P³) unit (Ho, 2020). Qualitatively, interview themes such as "making mathematics easier and more enjoyable," "productive struggle with complex tasks," and "developing problem-solving and communication skills" echo these patterns, as students described how repeated drills, hands-on group work, and portfolio tasks helped them remember procedures, explain their thinking, and persist with challenging problems (Hiltrimartin, & Pratiwi, 2025).

At the same time, the qualitative data nuance the largely positive quantitative profile by highlighting specific challenges for example, students' reports of difficult items and numerous drills align with items where fewer students selected "every or almost every lesson" and with slightly higher standard deviations on some activities, suggesting that not all learners experienced P-cube in the same way (Teig, Nilsen, & Hansen, 2024). The convergence between high reported engagement and performance and students' own descriptions of enjoyment, increased confidence, and skill development supports the interpretation that P-cube functioned as an engaging assessment context for this class, while the noted challenges point to areas where the design could be adjusted in future implementations (Hakim, Syamsurianti, Zaini, Bahri, & Suardi, 2025).

The quantitative and qualitative findings indicate that students did well on P-cube (P³) Practice and Participate tasks. They also reported greater engagement, confidence in procedures, and appreciation for group work. For example, classes with higher mean Participate task scores were those where interviewees most frequently described explaining solutions to peers and negotiating different strategies, whereas students who reported difficulty with the number of drills tended to be from groups with more variable Practice scores. These points of convergence and elaboration suggest that the thematic results aid in the interpretation of the patterns in the score distributions, rather than serving as a distinct line of evidence (Kiger, & Varpio, 2020).

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Because the study used a one-group post-implementation design without baseline or comparison data, these findings indicate high levels of fluency and procedural skills during the P-cubeunit but do not allow causal conclusions about improvement over time.

5. CONCLUSION

With the implementation of P-cube (P^3) assessment method in the class being incorporated in the topic about Polynomial Functions, the Grade 10-A students of Libertad National High School exhibited high levels of mathematical fluency and procedural skills, and described P-cube as a helpful and engaging way to practice and apply mathematical procedures. Positive responses were obtained from the students and only few statements gathered negative responses (Amerstorfer, & Von Münster-Kistner, 2021). The interview of the students revealed responses relating to the lessons they learned during the implementation of P-cube (P^3). Students learned that mathematics can be easily learned through engaging activities. Challenges were also met during the implementation process. Some challenges mentioned were the numerous drills and hands-on activities and difficulty of some problems to be solved. However, they had mentioned that their problem-solving, communication, and creativity skills were developed. P-cube (P^3) can be implemented in the future as this assessment method is fun and engaging according to the students. Students were having fun while they were learning about Polynomial Functions (Adeoye, & Jimoh, 2023). With the results obtained in this study, P-cube (P^3) as an authentic assessment method can also be implemented in other subjects.

Taken together, the high levels of performance in Practice and Participate tasks and students' reports of increased problem-solving, communication, and creativity indicate that P-cube (P^3) can translate sociocultural ideas about guided practice, mathematical discourse, and reflective portfolio work into a regular Grade 10 classroom.

For classroom practice, these results suggest that a manageable combination of daily individual drills (Practice), structured small-group activities (Participate), and a simple portfolio requirement (Portfolio) is a feasible way to support students' mathematical fluency and procedural skills in a resource-constrained public school.

Within the one-group post-implementation design, the study provides a descriptive profile of students' fluency, procedural skills, and perceptions under full P-cube (P^3) implementation, but it does not estimate gains relative to traditional assessment approaches.

5.1 Limitation of the study

This study has several limitations that should be considered when interpreting the findings. First, the sample consisted of one intact Grade 10 class from a single public school, selected through purposive sampling, so the results are context-specific and not statistically generalizable. Second, the one-group post-implementation design, without a pretest or comparison group, limits internal validity and does not rule out alternative explanations such as prior ability, teacher effects, or novelty of the assessment. Third, the measures of mathematical fluency and procedural skills focused on self-report questionnaire responses and scores from P-cube (P^3) activities within a two-week unit on polynomial functions, so the study did not examine conceptual understanding, transfer to other topics, or long-term retention. Finally, the implementation was conducted by a single teacher, which may influence how P-cube (P^3) functions in other classrooms and school contexts.

5.2 Implications for Practice and Recommendations

The findings of this study have several implications for classroom practice and for school-level decision making. Given that students in a resource-constrained public school reported high levels of mathematical fluency and procedural skills and described P-cube (P^3) as engaging and helpful, the assessment approach appears practically viable for regular Grade 10 mathematics instruction, within the limits of the design. At the same time, the challenges students reported, such as the difficulty of some problems and the number of drills and activities, point to areas where P-cube (P^3) can be refined to better support diverse learners.

For classroom teachers, the results suggest integrating a manageable form of P-cube (P^3) by combining short daily individual drills on current topics (Practice), structured small-group problem-solving tasks at least once or twice

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per week (Participate), and a simple portfolio where students compile selected work, reflections, and group outputs (Portfolio). To address differentiation and workload, teachers should carefully calibrate the number and difficulty of drills and hands-on tasks so that students experience productive challenge without being overwhelmed, for example by providing tiered problem sets and optional enrichment items. For school leaders and department heads, it is recommended to provide time and support for teachers to collaboratively design Practice, Participate, and Portfolio tasks aligned with the curriculum and to share samples of student work as models of how P-cube (P^3) can be implemented in different classes. Finally, for professional development, P-cube (P^3) can be used as a focal point for training on sociocultural and authentic assessment approaches, emphasizing how guided practice, mathematical discourse, and portfolio-based reflection can be integrated into routine lessons rather than added as extra tasks.

5.3 Suggestions for Future Research

Future research could conduct pretest–posttest or quasi-experimental studies that compare P-cube(P^3) with more traditional assessment methods in order to estimate its impact on students' mathematical fluency, procedural skills, and conceptual understanding. It would also be valuable to replicate the implementation of P-cube(P^3) with multiple classes, schools, and grade levels to examine whether similar results emerge across different teachers and contexts. Further studies should include additional data sources, such as performance-based problem-solving tasks, standardized test subscores, or classroom observations, to triangulate questionnaire responses and activity scores. In addition, researchers can explore the use of P-cube(P^3) in other mathematics topics, such as rational expressions, systems of equations, or geometry, and over longer periods to investigate transfer and the durability of effects.

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